parameter $k_1 = \delta \cot\alpha$. For both types of flow discussed, the governing equations in the inner region can be written in terms of the perturbation velocity potential as

$$\frac{\partial^2\phi}{\partial Y^2} + \frac{\partial^2\phi}{\partial Z^2} = 0$$

$$P - P_\infty = -\delta \cot\alpha \frac{\partial\phi}{\partial X} - \frac{\partial\phi}{\partial Y} - \frac{1}{2}\left(\frac{\partial\phi}{\partial Y}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial Z}\right)^2$$

where

$$P_\infty = 1/\gamma M_\infty^2 \sin^2\alpha$$

Note that these equations can also be obtained with slender-wing and slender-body theory.[6] It is concluded that the normal-force and axial-force coefficients for incompressible flow and for subsonic and supersonic flows with small values of the crossflow Mach number can be expressed as

$$C_N = k_1 \sin^2\alpha \hat{C}_N(k_1, M_\infty) \qquad C_A = \delta k_1^2 \sin^2\alpha \hat{C}_A(k_1, M_\infty)$$

As slender-wing and slender-body theory shows, the dependence of C_N on M_∞ is weak and can be neglected for potential flow. However, the dependence of C_A on M_∞ is not negligible.

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Similarity Rule for Sidewall Boundary-Layer Effects in Airfoil Testing

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Introduction

THE sidewall boundary-layer interference in testing of airfoils in wind tunnels has recently been the subject of considerable attention. Earlier methods to account for the sidewall effects were based on the vorticity model proposed by Preston.[1] However, following recent experimental observations made in the ONERA[2] tunnel, Barnwell,[3,4] and Winter and Smith[5] have independently proposed theories based on the changes in the sidewall boundary-layer thickness due to the airfoil flowfield. In the form proposed by Barnwell, a factor similar to the Prandtl-Glauert rule was suggested to account for the sidewall boundary-layer effects. This was later extended to transonic speeds by Sewall[6] by using the von Kármán similarity rule. In this Note, an alternative simpler form of the similarity rule is presented by considering the sidewall boundary layer to cause changes in both the airfoil thickness and the freestream Mach number. This approach, within the small-disturbance approximation, encompasses both the methods of Barnwall and Sewall and, hence, can be used from low speeds to transonic speeds.

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Analysis

For the flow over an airfoil mounted between the walls of a two-dimensional wind tunnel of width b, the sidewall boundary-layer effects can be represented in a simplified manner by the small-disturbance equation[4,6,7]

$$(1 - M^2 + k)\left(\frac{\partial u}{\partial x}\right) + \left(\frac{\partial v}{\partial y}\right) = 0 \tag{1}$$

where x, y refer to the streamwise and normal coordinates, u, v are the perturbation velocities, and M is the local Mach number. In arriving at Eq. (1), it was assumed that the equivalent flat-plate Reynolds number for the sidewall boundary layer was much larger than the airfoil chord Reynolds number, and the changes in the boundary-layer thickness introduced cross-flow velocities that varied linearly across the tunnel width. The parameter k is nearly constant and is given by the values of the undisturbed sidewall boundary-layer displacement thickness (δ^*) and the shape factor H.

$$k = (2\delta^*/b)(2 + 1/H - M^2) \tag{2}$$

Introducing the coordinate transformation $\xi = x$ and $\eta = y(1+k)^{1/2}$, and the velocity potential ϕ, Eq. (1) can be reduced to an equivalent two-dimensional flow represented by

$$(1 - M_e^2)\phi_{\xi\xi} + \phi_{\eta\eta} = 0 \tag{3}$$

where $M_e = M/(1+k)^{1/2}$ is the local Mach number in the equivalent flow. If the freestream velocity is U_∞ and the airfoil thickness is τ, the corresponding boundary condition on the airfoil surface is given by

$$(\phi_\eta)_{\eta=0} = U_\infty \tau (1+k)^{-1/2} f'(x/c) \tag{4}$$

where $f(x/c)$ represents the airfoil shape. From the transformed boundary condition (4), it follows that, in the equivalent flow represented by Eq. (3), either the freestream velocity or the airfoil thickness ratio can be considered to be reduced by a factor of $(1+k)^{-1/2}$. For subsonic flow, Eq. (3) can be linearized by approximating M by the freestream value M everywhere. The corresponding freestream Mach number M_c in the equivalent two-dimensional flow will be $M_c = M_\infty/(1+k)^{1/2}$.

In transonic flow, the Mach number is reduced everywhere by the factor $(1+k)^{-1/2}$. However, for both subsonic and transonic flows, if the freestream velocity in the equivalent flow is U, it follows from Eq. (4) that the equivalent flow corresponds to flow over a thinner profile with a thickness ratio of $\tau_e = \tau/(1+k)^{1/2}$. This equivalent two-dimensional flow can be related to a number of other two-dimensional flows by using the high-speed similarity rules, and the results of Refs. 3 and 6 can be obtained as particular cases.[7] For subsonic flow, if the freestream velocity in the equivalent flow is U_e $[= U_\infty/(1+k)^{1/2}]$, the corresponding pressure coefficient $C_{p,e}$ will be

$$C_{p,e} = -2[\phi_\xi / U_e]_{\eta=0} \quad (5)$$

Equation (5) implies that the flow in the wind tunnel at Mach number M_∞ will be equivalent to that in a two-dimensional flow at Mach number M_c on the same profile, with the pressure coefficients increased by a factor of $(1+k)^{1/2}$ over the wind-tunnel value.

For transonic flow, the pressure coefficients cannot be scaled in direct proportion to the change in the airfoil thickness because of the nonlinearity of Eq. (3). The equivalent two-dimensional flow at Mach number M_c on an airfoil of the thickness τ_e can be related to another flow at Mach number $\bar{M}_c$ on an airfoil of thickness τ by using the von Kármán similarity parameter. This procedure leads to the results of Ref. 6, which can be expressed as

$$(1 - M_\infty^2 + k)/(1 - \bar{M}_c^2) = (M_\infty / \bar{M}_c)^{4/3} \quad (6)$$

and

$$C_{p,c} = C_{p,m}(M_\infty / \bar{M}_c)^{2/3} \quad (7)$$

where $\bar{M}_c$ refers to the new corrected Mach number, $C_{p,c}$ the corrected pressure coefficient, and $C_{p,m}$ the measured value of the pressure coefficient. Equation (7) can be written in a convenient form as

$$\bar{M}_c/(1 - \bar{M}_c^2)^{3/4} = [M_c/(1 - M_c^2)^{3/4}](1+k)^{-1/4} \quad (8)$$

For small k, the corrected values can be expressed as

$$\bar{M}_c \approx M_c = M_\infty (1+k)^{-1/2} \quad (9)$$

and

$$C_{p,c} \approx (1+k)^{1/3} C_{p,m} \quad (10)$$

Results and Discussion

From the preceding analysis, it follows that the wind-tunnel flow over an airfoil with sidewall boundary layers can be represented by an equivalent two-dimensional flow with the test Mach number and the airfoil thickness reduced by a factor of $(1+k)^{-1/2}$. However, if the airfoil thickness in the equivalent flow is assumed to be the same as the tested airfoil, the measured coefficients are to be increased by a factor of $(1+k)^{1/3}$ at transonic speeds.

The use of similarity rules permits construction of several equivalent two-dimensional flows that are equally valid. One particular case corresponds to the subsonic linearized flow considered in Refs. 3 and 8. This is obtained by considering the pressure coefficient in the new flow to be the same as in the equivalent flow but at a different corrected Mach number M_{cb} on an airfoil of thickness τ. Then, by the Prandtl-Glauert rule, M_{cb} and M_c are related by

$$(1 - M_{cb}^2)/(1 - M_c^2) = (\tau/\tau_e)^2 \quad (11)$$

or

$$M_{cb}^2 = M_\infty^2 - k \quad (12)$$

With this approach, the corrected Mach number M_{cb} is not defined for $M_\infty^2 < k$. This anomaly is due to the fact that the effects of compressibility and of sidewall boundary layers are of opposite nature. That is, for the pressure coefficient to remain the same, the airfoil thickness in the new flow being larger than τ_e in the equivalent vlow, the Mach number has to be reduced. Hence, it appears that the appropriate corrected Mach number is given by M_c rather than by that given in Eq. (12), according to Refs. 3 and 8. A comparison of the present correction for the Mach number with other methods is shown in Fig. 1. The present method shows a continuous increase in the correction from incompressible to transonic speeds. At higher Mach numbers, the difference between the present simplified correction and that of Ref. 6 is not significant.

The large difference at low Mach numbers between Eq. (12) due to Barnwell and other results is due to the fact that the Barnwell method was proposed primarily to treat "lift correction" rather than "Mach number correction." This was demonstrated by Barnwell,[3] who correlated the lift measurements on an airfoil obtained in the ONERA tunnel with different sidewall boundary-layer thicknesses.[2] Still, it is interesting to note that the Mach number correction given by Eq. (12) is close to the results of other methods at transonic Mach numbers. In the form proposed by Sewall,[6] both the lift and test Mach number are corrected. For small values of the sidewall boundary-layer displacement thickness, all of the different approaches give nearly the same correction at higher Mach numbers.

In the form proposed, the equivalent flow has been considered without directly invoking the similarity laws. This method of representing the effect of a sidewall boundary layer as causing changes in the airfoil thickness and the test Mach number is applicable from low speeds to high Mach numbers. At low speeds, the thickness effect will be dominant; at transonic speeds, the Mach number correction is important since the flow is sensitive to thickness distribution rather than the thickness. The proposed transformation provides a direct scaling for the local Mach number when testing with different sidewall boundary-layer thicknesses. This is demonstrated in Fig. 2 by correlating the experimental data on a supercritical airfoil[9] for two different values of the sidewall boundary-layer thicknesses. The proposed transformation gives good correlation, particularly on the upper surface of the airfoil. It may be noted that the present correction and the correction proposed in Refs. 3 and 6 are applicable only when the airfoil chord is large so that the sidewall boundary-layer effect is nearly one-dimensional. For short chord or large aspect ratio models, the sidewall boundary-layer effects are likely to be much smaller, at least near the midspan region.

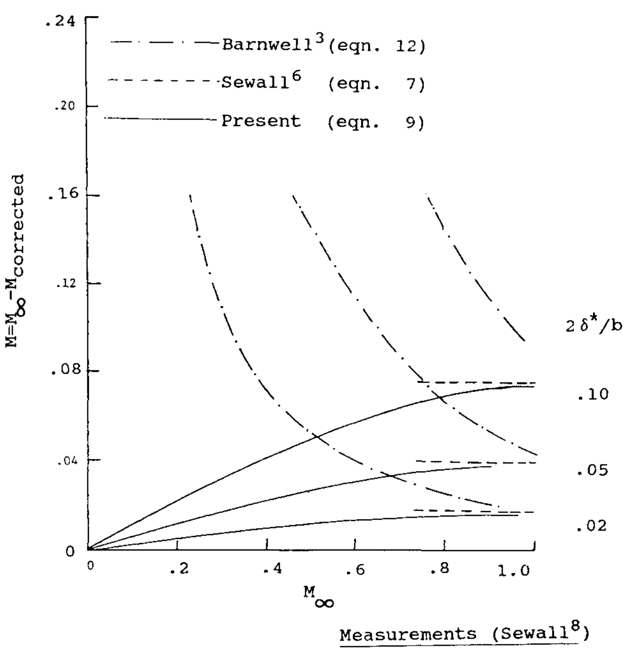

Fig. 1 Comparison of the results of the present similarity rule with other methods for correction to the freestream Mach number.

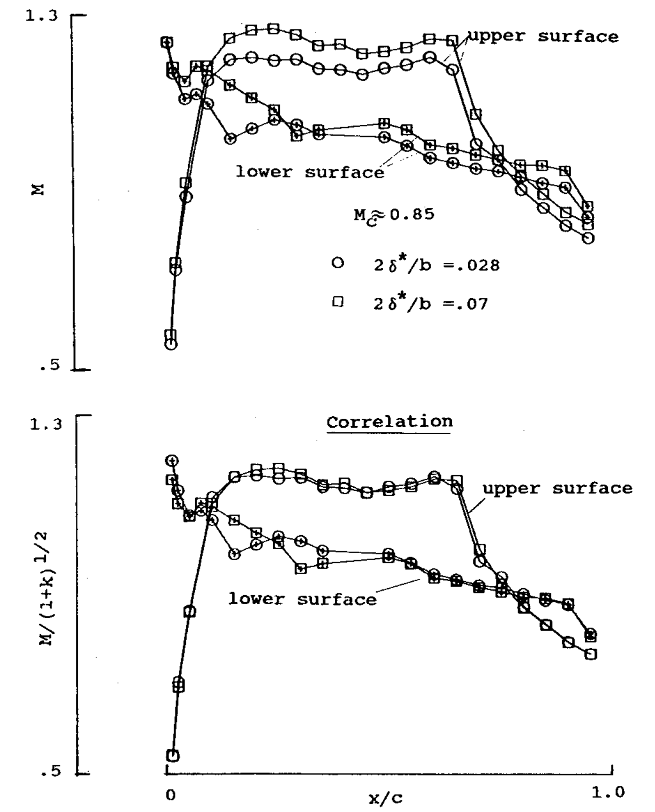

Fig. 2 Correction of local Mach number measurements on a supercritical airfoil at zero incidence with different sidewall boundary-layer thicknesses.[9]

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Comparison of Five Methods for Determination of the Wall Shear Stress

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Introduction

THE wall shear stress τ_w is a crucial parameter for determining transport of mass and energy in ducts and the neighborhood of fixed walls and for determining the drag on constructions immersed in fluid flow. Also, since most of the universal scaling laws for turbulent boundary layers involve the friction velocity, $v_* = (\tau_w/\rho)^{1/2}$, it is of primary importance to have accurate methods for measuring τ_w. The review by Winter[1] explores most of the methods available for direct and indirect measurement of τ_w in turbulent boundary layers. It is considered to be difficult, time consuming, and costly to accomplish accurate direct measurements of the local wall shear stress by using methods involving, for example, floating elements. In the present work, five methods are used to determine τ_w, expressed through the dimensionless skin-friction coefficient c_f, as

$$c_f = 2\tau_w/(\rho U_\infty^2) = 2(v_*/U_\infty)^2 \tag{1}$$

The methods are of the indirect type and are based on the shape of the mean velocity profile.

In order to isolate the variables as far as possible a two-dimensional zero pressure gradient turbulent boundary-layer (TBL) flow with well-defined boundary conditions was established as the test case for comparison of the methods for determination of τ_w. Effort was made in order to achieve what Coles[2] classifies as a "normal" two-dimensional TBL as the test case. A two-dimensional TBL may easily be distorted by factors as three-dimensional flow effects, a poor choice of the tripping device and high values of the freestream turbulence intensity. The effect of a high value of the freestream turbulence level is to affect the mean velocity profile in the outer part of the boundary layer. Blair[3] found that the mean velocity profile is not influenced by freestream turbulence when the freestream turbulence level is less than about 1%. A tripping strip is often used to promote a stable transition to a TBL. A poor choice of this device may cause disturbances from which the TBL may be very slow to recover.[4,5] Three-dimensional effects such as small mean crossflow components may seriously influence the local skin friction when compared to two-dimensional theory.[6,7] Such small mean crossflow components are often hardly measurable. There are, however, several parameters characterizing the development of a normal two-dimensional TBL, which will be reported in the following sections.

Experimental System

The turbulent boundary layer investigated develops on the flat floor in the 500 × 1000 mm test section of a closed-return wind tunnel where the velocity range is 1–40 m/s and the

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